

Measuring mean densities of δ Scuti stars with asteroseismology

Theoretical properties of large separations using TOUCAN

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ABSTRACT

Aims. We aim at studying the theoretical properties of the regular spacings found in the oscillation spectra of δ Scuti stars.

Methods. We performed a multi-variable analysis covering a wide range of stellar structure and seismic properties and model parameters representative of intermediate-mass, main sequence stars. The work-flow is entirely done using a new Virtual Observatory tool: TOUCAN (the VO gateway for asteroseismic models), which is presented in this paper.

Results. A linear relation between the large separation and the mean density is predicted to be found in the low frequency frequency domain (i.e. radial orders spanning from 1 to 8, approximately) of the main-sequence, delta Scuti stars' oscillation spectrum. We found that such a linear behavior stands whatever the mass, metallicity, mixing length, and overshooting parameters considered in this work. The intrinsic error of the method is discussed. This includes the uncertainty in the large separation determination and the role of rotation. The validity of the relation found is only guaranteed for stars rotating up to 40% of their break-up velocity. Finally, we applied the diagnostic method presented in this work to five stars for which regular patterns have been found. Our estimates for the mean density and the frequency of the fundamental radial mode match with those given in the literature within a 20% of deviation.

Conclusions. Asteroseismology has thus revealed an independent direct measure of the average density of δ Scuti stars, analogous to that of the Sun. This places tight constraints on the mode identification and hence on the stellar internal structure and dynamics, and allows a determination the radius of planets orbiting around δ Scuti stars with unprecedented precision. This opens the way for studying the evolution of regular patterns in pulsating stars, and its relation with stellar structure and evolution.

Key words. asteroseismology, stars: evolution, stars:variables: delta scuti, stars: oscillations (including pulsations), stars: interiors, virtual observatory tools

1. Introduction

Nowadays, thanks to the asteroseismic space missions like *MOST* (Walker et al. 2003), *CoRoT* (Baglin 2003), and *Kepler* (Gilliland et al. 2010), the study of the intrinsic variability of A- and F-type stars is living a revolution. In particular, the large number of modes detected and the variety of frequency domain covered has given rise to the so-called *hybrid* phenomenon, which imposes a revision of the current observational instability strips of δ Scuti and γ Dor stars (see Uytterhoeven et al. 2011, and references therein). Those stars have been found to be located over the entire γ Dor and δ Scuti instability strips, which implies that a review of their pulsation mechanisms is necessary to supplement (or even substitute) the convective blocking effect and the κ mechanisms, respectively. From the theoretical side, based on stellar energy balance studies, Moya & Rodríguez-López (2010) concluded that δ Scuti stars are able to excite hundreds of pulsation modes, and whose accumulated pulsation energy is not large enough to destroy their hydrostatic equilibrium. Likewise, the richness found in the oscillation spectrum of these stars is interpreted by Kallinger & Matthews (2010) as non-radial pulsation superimposed on granulation noise (correlated noise).

In this work we focus on δ Scuti stars which are intermediate-mass (i.e., from 1.4 to 3 M_{\odot} , approximately), mainly in the main sequence and the sub-giant branch, but also in the pre-main sequence. Their spectral types range from A2 to F5, and their luminosity classes go from IV to V, approximately. This locates them at the lower part of the Cepheid's instability strip. Their pulsations are mainly driven by the so-called κ mechanism (see reviews by Breger 2000; Handler 2009), showing radial and non-radial oscillation modes excited in a frequency domain ranging from a few tens of μHz up to about 800 – 900 μHz (see e.g. Poretti et al. 2011).

The interpretation of the oscillation spectra of these stars has never been an easy task. Due to the complexity of their oscillation spectra the identification of detected modes is often difficult. A unique mode identification is often impossible and this hampers the seismology studies for these stars. Additional uncertainties arise from the effect of rapid rotation, both directly on the hydrostatic balance in the star and, perhaps more importantly, through mixing caused by circulation or instabilities induced by rotation (see e.g. Zahn 1992).

However some decades ago, this scenario started to change thanks to the detection of regular patterns in the detected frequencies. These patterns were observed in the low radial order modes (mixed modes) frequency domain, in which main-sequence classical pulsators show the maximum of their os-

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cillation power, i.e. between 80 and 800 μHz , hereafter intermediate frequency domain. There, regularities similar to those found in solar-like stars were not expected (for a review on this topic, see Goupil et al. 2005). Thanks to great efforts made in long ground-based multi-site campaigns, Handler et al. (1997) found regularities (near 26 μHz) in the oscillation spectrum of the young δ Scuti star CD-24 7599, which contained 13 frequencies. Those regularities were found using a Fourier transform technique. Also Breger et al. (1999) found a regular spacing close to 46 μHz in the oscillation spectrum of the δ Scuti star FG Vir, composed of 24 frequencies, searching for regular spacings using histograms of frequency differences, instead of Fourier transform. Later on, they re-studied the star using data spanned in ten years (Breger et al. 2009), in which the number of detected frequencies increased to 68, approximately. The frequency spacing found was attributed to an observed clustering of certain non-radial modes around radial ones. This clustering was explained by the higher probability of certain $\ell = 1$ modes (the so-called trapped modes Dziembowski & Krolikowska 1990) to be excited to observable amplitudes than other modes.

Thanks to the different space missions providing asteroseismic data, similar studies were undertaken, with much more frequencies detected with very high precision. In 2007, Matthews (2007) found 88 frequencies in the oscillation spectrum of the δ Scuti star HD209775 observed by *MOST*. In that work, a regular spacing of 50 μHz approximately was obtained using histograms of frequency differences. This was assumed to be a large separation, although no physical explanation to support this was given. Recently, regular patterns were also found in the oscillation spectra of δ Scuti stars observed by *CoRoT* (García Hernández et al. 2009; Mantegazza et al. 2012; García Hernández et al. 2013) and *Kepler* (Hernández et al. 2013), and the hypothesis of identifying them with the large separation has been reinforced.

Theoretically large separation is expected to grow from low to high radial orders. In contrast to the well-defined plateau shown by the large separation in the high radial order domain (the so-called asymptotic regime, see e.g. Antoci et al. 2011; Zwintz et al. 2011), it forms a quasi-periodic structure at low radial order regime (see Fig. 4 of García Hernández et al. 2009). In that work, it was shown that the standard deviation of such an structure is roughly 2.5 μHz , and the distance between the mean value of this structure and the high order regime is of the order of 5 μHz .

The main questions we intend to answer in the present work are: What are the physical properties of the periodicities observed in the low frequency domain of δ Scuti stars? Do they have similar properties to those found in solar-like stars?

The above questions are tackled by analyzing the properties of the predicted regularities over a large collection of models and physical variables. To do so, we developed a virtual observatory tool, TOUCAN¹, designed to easily handle stellar and seismic models, examine their properties, compare them with observational data and find models representative of the studied star(s). The tool is presented in this paper.

2. The method

We examine a dense sample of asteroseismic models representative of A-F main-sequence stars, i.e. covering the corresponding area in the HR diagram where classical pulsations for these

Table 1. Ranges of the four parameters used to construct the current model dataset representative of intermediate-mass stars.

Parameter	Lowest	Highest	Step
M/M_{\odot}	1.25	2.20	0.01
$[Fe/H]$	-0.52	+0.08	0.20
α_{ML}	0.50	1.50	0.50
d_{OV}	0.10	0.30	0.10

stars are expected. Such a modeling approach requires an efficient computing procedure as well as the capability of managing and analyzing large sets of models. For the former, we performed most of the computation using the GRID computing service provided at IAA-CSIC as one of the nodes of the *Ibergrid* virtual organization.

For the latter, we have developed TOUCAN, a VO tool able to easily compare different and heterogeneous collections of asteroseismic models (equilibrium models and their corresponding synthetic oscillation spectra). Details about the work-flow followed and/or the different TOUCAN services can be found in Appendix B.4 and B.3, respectively. The model collection used in the present work is described in the next section.

2.1. The grid of models

We constructed a model collection composed of approximately $5 \cdot 10^5$ models of intermediate-mass stars (namely δ Scuti and γ Dor stars). For the sake of homogeneity and precision of the asteroseismic mode sample, models were computed following the prescriptions suggested by ESTA/CoRoT² working group (Moya et al. 2008; Lebreton et al. 2008).

The equilibrium models were computed with the evolutionary code CESAM (Morel 1997; Morel & Lebreton 2008). Oscillation frequencies were computed with GraCo (Moya et al. 2004; Moya & Garrido 2008), which uses the perturbative approach, to provide adiabatic and non-adiabatic quantities related to pulsation and includes the convection-pulsation interaction using the Time Dependent Convection theory (TDC, Dupret et al. 2005).

The model grid is composed of evolutionary tracks, evolved from ZAMS up to the sub-giant branch. Each track contains about 250 equilibrium models with their corresponding oscillation spectra. The equilibrium models were computed varying four quantities: two global stellar parameters, mass and metallicity (M and $[Fe/H]$), and two modeling parameters often used in asteroseismology, the convection efficiency, $\alpha_{ML} = l/H_p$, where l is the mixing length and H_p is the pressure scale height, and the overshoot parameter $d_{OV} = l_{ov}/H_p$ (l_{ov} being the penetration length of the convective elements). The range of variation for each parameter in this dataset is listed in Table 1. The physical parameters of the models were chosen as the typically adopted for δ Scuti stars.

Although δ Scuti stars have also been observed in the pre and post-main sequence evolutionary stages, the present work focuses on the main sequence (i.e. hydrogen-burning phase in the convective core) where the low-order periodicity has been found (see previous section).

¹ <http://svo.cab.inta-csic.es/theory/sisms3/index.php>

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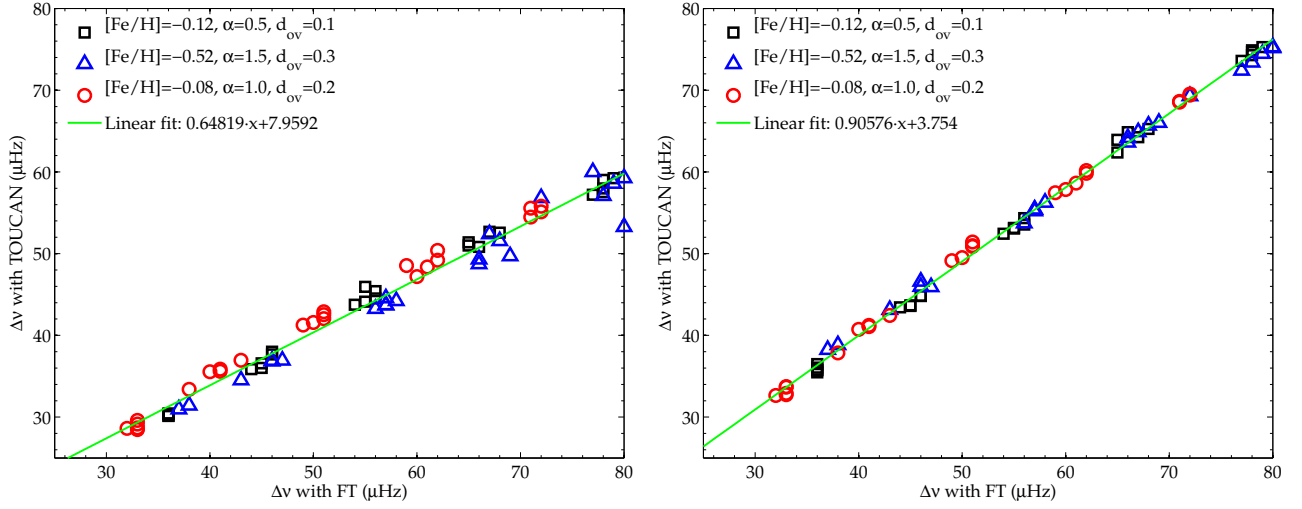


Fig. 1. Left panel: spacings found using the FT method are compared to large separation values computed with TOUCAN in the range of the observed modes for δ Scuti stars. Right panel: Similar to left panel, but only p modes, with radial order $n \geq 1$ were considered in the calculation of TOUCAN $\Delta\nu$. Spacings obtained with the FT method and TOUCAN computations are only equivalent under the conditions adopted in the right panel. Colors are only available in the online version of the paper.

2.2. Large frequency separation

The frequency difference defined as

$$\Delta\nu_\ell = \nu_{n,\ell} - \nu_{n-1,\ell} \quad (1)$$

is known as a large separation, where $\nu_{n,\ell}$ is the frequency of a mode with radial order n and degree ℓ , whatever the azimuthal order m . This frequency difference is nearly constant for p modes in the asymptotic regime, and obeys, in a first approximation, the following physical dependence (Tassoul 1980; Gough 1990):

$$\Delta\nu = 2 \left[\int_0^R \frac{dr}{c_s} \right]^{-1} = \tau^{-1} \quad (2)$$

where c_s is the sound speed and τ is the acoustic time between 0 and R (i.e. the stellar center and surface, respectively) which define the stellar region in which the oscillation mode is propagating.

TOUCAN calculates $\Delta\nu$ everywhere in the theoretical oscillation spectrum, for any combination of angular degree (up to $\ell = 3$), frequency and radial order domains. In practice, the calculation is performed in two steps. First, for a given angular degree ℓ , i.e.

$$\Delta\nu_\ell = \frac{1}{N_n} \sum_{n=n_j}^{n_{k-1}} (\nu_{n+1} - \nu_n), \quad j < k \quad (3)$$

where n is the radial order, and $N_n = n_k - n_j$ is the number of frequencies found in the interval $[n_j, n_k]$ for the given ℓ . Then TOUCAN calculates the average of $\Delta\nu_\ell$ over all the angular degrees calculated in the model, i.e.,

$$\Delta\nu = \langle \Delta\nu_\ell \rangle = \frac{1}{N_\ell} \sum_{\ell=\ell_i}^{\ell_j} \Delta\nu_\ell \quad (4)$$

where $N_\ell = \ell_j - \ell_i$ is the total number of angular degrees considered. Here we are interested in studying the physical properties of such separations for the intermediate frequency domain. But, before analysing the TOUCAN results, we need to take into account some considerations with respect to the rotation and the presence of mixed modes.

2.3. Rotation and mixed modes

The A-F type stars are generally fast rotators, and rotation effects on both the stellar structure and the oscillation frequencies cannot be neglected, particularly those caused by stellar distortion (Soufi et al. 1998; Suárez et al. 2006), even for $m = 0$ modes with which $\Delta\nu$ are calculated in the present work (Suárez et al. 2010).

Rotation effects on oscillations is commonly taken into account through the perturbation approximation (Dziembowski & Goode 1992), which is limited to slow-to-moderate rotation, i.e. small stellar deformations (see e.g. Suárez et al. 2005; Reese et al. 2006, for semi-empirical and theoretical studies of the limitations of the perturbation approximation). Therefore, for moderate-to-rapid rotators a non-perturbative approach for the calculation of the oscillation modes on a deformed star becomes necessary (e.g. Lignières et al. 2006). However, nowadays this calculation is available for polytropic models and for some more realistic fully deformed 2D stellar models on the ZAMS based on the self-consistent field (SCF) method (Jackson et al. 2005; MacGregor et al. 2007; Reese et al. 2009) which considers the models chemically homogenous with angular velocity assumed to be dependent only on the distance from the axis of rotation. Furthermore, these latter models together with the calculation of non-perturbative oscillations require a significant amount of computing resources as well as time of computation. Therefore the use of a proper modeling for rapidly rotating stars would be unpractical for the present work.

On the other hand, since periodicities are indeed observed, it might be concluded that rotation effects are not sufficient to break the regularities. Indeed, non-perturbative calculations of the oscillation spectra for rapidly rotating polytropic models indicate that as rotation increases, the asymptotic structure of the non-rotating frequency spectrum is replaced by a new form of organization (Reese et al. 2008, 2009). This new mode frequency organization also exhibits regular structures, including the large separation (see Lignières et al. 2010, for a recent theoretical work on regular patterns in rapidly-rotating stars) whose varia-

tion (normalized by the density) from the non-rotating case is negligible (Lignières et al. 2006). Furthermore, calculations of non-perturbative oscillation frequencies on SCF models shows a maximum variation of the large separation of around $2.3 \mu\text{Hz}$ for stars rotating up to 40% of the keplerian velocity (Reese, private communication), which is small compared with the precision with which the periodicities are predicted at both low- and high-frequency domains ($10 \mu\text{Hz}$ approx., see e.g. Mantegazza et al. 2012). Considering all the above theoretical arguments, we are allowed to use non-rotating models for the present study.

This thus reinforces the hypothesis that identifies the observed periodicities with the large separation, as well as the hypothesis that these appear in a new distribution of modes predicted by the non-perturbative theory. In order to check those hypothesis, it is necessary to analyze the oscillation spectra of many stars with different rotational velocities. Thanks to space missions like *CoRoT*, *Kepler*, or *PLATO*, this study can be tackled in the near future.

Another issue that might hamper the detection of periodicities, in particular, of the large separation, is the presence of mixed modes. This phenomenon is implicitly considered in this work since the selected models cover all the main sequence. In contrast, this cannot be properly studied within the non-perturbative approach, since both the polytropic and the SCF models in ZAMS are not expected to properly show the avoided crossing phenomenon. In order to properly understand how avoided-crossing may affect the detection of periodicities, the present analysis should be done for a more evolved SCF model (work in progress).

On the other hand, all these results were obtained assuming no dependence with the visibility of the modes which may introduce artificial periodicities that may potentially be confused with the large separation. Recently, this has been studied in the framework of the non-perturbative theory. Specifically, it has been found (Reese et al. 2013) that acoustic modes with the same $(\ell, |m|)$ values tend to have similar amplitude ratios, although this effect is not systematic. Since the global visibility of the modes decreases while the mode degree increases, we do not expect this effect to affect the large separation determination significantly. In any case, a work on the influence of mode visibilities in the study of periodicities for rapid rotators is currently ongoing.

2.4. Large separations computation

In GH09, regular spacings were found using the Fourier transform (hereafter, FT). Here we study whether this technique and the average of the periodicity used by TOUCAN are equivalent.

To do so, we used a subset of asteroseismic models within typical values for T_{eff} and $\log g$ of δ Scuti stars with their corresponding uncertainties. For completeness, we considered the following extreme combinations of the physical parameters listed in Table 1 divided into three sets:

- #1 $[\text{Fe}/\text{H}] = -0.12$, $\alpha_{ML} = 0.5$, and $d_{OV} = 0.1$
- #2 $[\text{Fe}/\text{H}] = -0.52$, $\alpha_{ML} = 1.5$, and $d_{OV} = 0.3$
- #3 $[\text{Fe}/\text{H}] = 0.08$, $\alpha_{ML} = 1$, and $d_{OV} = 0.2$

For each set of parameters, we selected five models covering the extremes values (four models) and the central value (one model) of effective temperature and gravity considered in the selected model subset.

For each set of five models, we computed the large separation using both methods in the intermediate frequency domain, i.e., no restrictions in the radial orders but only in the

frequency range were considered in order to mimic the observations. Independently of the combination of model parameters the relation between the large separations obtained with the FT method and those obtained by TOUCAN was found to be linear (Fig. 1, left panel).

On the other hand, the corresponding linear fit does not have a slope equal to one, which would be expected if the two methods were equivalent. In order to understand this, we first attempted to explain such discrepancies by the presence of gravity modes (g modes). However, when removing g modes in TOUCAN computations (not in the FT ones), the slope of the trend was improved but not equal to one, neither. We then considered the well-known ratio between the period of the fundamental radial mode and the period of its first overtone (which is 0.77 for instance, for population I, main sequence stars). This ratio, which is constant for each δ Scuti star, implies a relation between these two radial modes which does not necessarily follows the average behavior of $\Delta\nu$. To avoid such a contribution in the calculation of the large spacings by TOUCAN, we also restricted the oscillation modes to those with a radial order $n \geq 1$.

With all these requirements, the FT and TOUCAN large separations are found to be formally equivalent (Fig. 1, right panel). These requisites were thus adopted throughout the whole work.

Note that we always used the same range of frequencies for computing the FT. This test emphasizes the robustness of the FT method to detect the large separation even when g modes and the fundamental radial mode are not discriminated. Detailed analysis of the slopes obtained in the two cases indicates that an uncertainty of 10% in the equivalence should be considered if other constraints, namely in the frequency range, are not taken into account.

3. Large separation vs. mean density

One of the main characteristic of large separations observed at high frequency (e.g. in solar-like stars) is that they are proportional to the stellar mean density through Eq. 2, which, together with another regularity, the so-called small separation, provides a direct measure of the mass, radius, and evolutionary stage of those stars (see e.g., Christensen-Dalsgaard 1988). It is thus worthy to investigate if such a dependence is also predicted at low frequencies, in particular in the frequency domain where δ Scuti stars pulsate.

To do so, we followed the workflow described in Appendix B.4 to select a set of models to work with. In particular, we considered all the models contained in the model database described in the previous sections, i.e. those whose parameters are described in Table 1. For the sake of simplicity, the study is restricted to models in the main sequence, that is, between zero-age to turn-off main sequence stages. In what regards the asteroseismic content, we adopted the prescriptions deduced in the previous section, i.e., we computed the large separation in the intermediate frequency domain using radial and non-radial modes with $n \geq 1$, and $\ell \leq 3$. Indeed, the presence of modes with $\ell > 3$, whose presence is suggested by spectroscopic studies (Mantegazza et al. 2012), would hamper the determination of the large separation. Since we are dealing with oscillation spectra obtained from photometry, and since the FT method (details in GH09) uses the modes with the largest amplitudes, it is plausible to consider that those modes correspond to $\ell = [0, 3]$ modes.

Analysis of the large separation predicted in the intermediate frequency domain as a function of the mean density reveals (Fig. 2) a clear relation which can be expressed mathematically

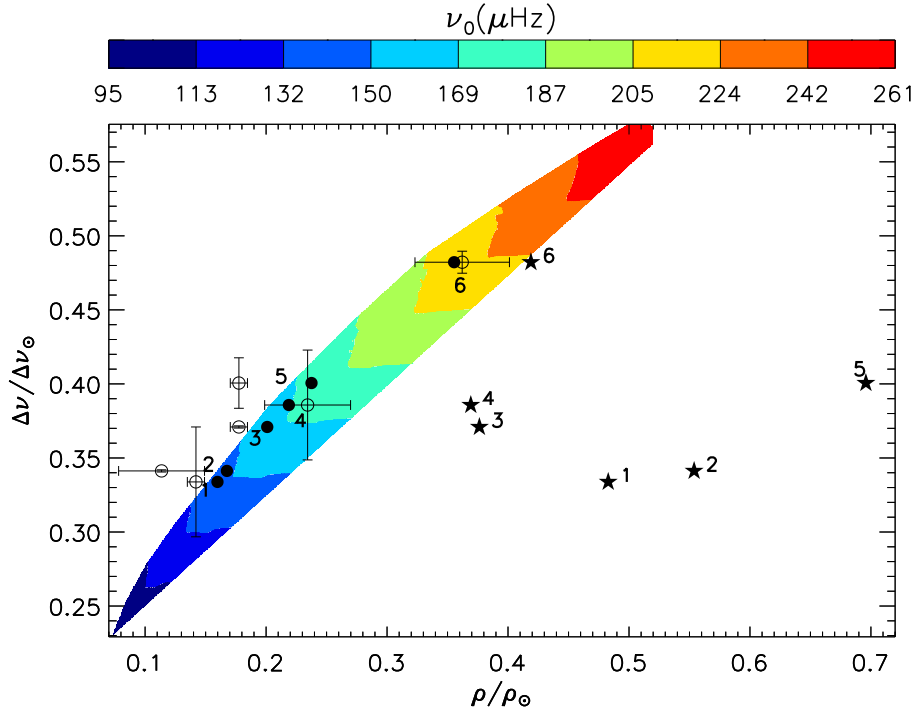


Fig. 2. Predicted large separation as a function of the mean density of the star, normalized to their solar values, $\Delta\nu_{\odot} = 134.8\mu\text{Hz}$ (Kjeldsen et al. 2008) and $\bar{\rho}_{\odot} = 1.48\text{gcm}^{-3}$ (Haynes et al. 2012), respectively. Contours in gray scale indicate the predicted frequency of the fundamental radial mode. Filled dots, empty dots, and star symbols represent mean densities found in this work, in the literature, and using Tingley’s calibration, respectively. Stars are labelled as in Table 2. For the sake of clarity, the error bars in star symbol estimates are omitted, since they are larger than the abscissa range. Colors are only available in the online version of the paper.

as

$$\Delta\nu/\Delta\nu_{\odot} = 0.776(\rho/\rho_{\odot})^{0.46}. \quad (5)$$

obtained by performing a linear regression of the complete set of models. This gives us an approximation of the overall behavior of the relation between $\Delta\nu$ and $\bar{\rho}$ in the intermediate frequency domain which is predicted here to be proportional to the mean density of the star to the power 0.46, which is quite close to $\Delta\nu \propto \rho^{1/2}$, predicted for high radial orders. This opens the way for studying the evolution of regular patterns in pulsating stars throughout the HR diagram, and its relation with stellar evolution. This also might provide new insights in the hybrid phenomenon, i.e. the yet unexplained excitation modes in a wide frequency range covering typically the γ Doradus, δ Scuti and even solar-like oscillation modes (Uytterhoeven et al. 2011).

We present no errors in the coefficients of the fitting because none of the theoretical $(\Delta\nu_i, \bar{\rho}_i)$ are independent from each other, and therefore regression error estimates are meaningless. Nevertheless it is possible to examine the domain of validity of the method. In Appendix A we provide estimates of intrinsic errors of the method and their relation with the domain of validity of the present method.

These results imply that Fig. 2 can be used as a powerful diagnostic tool for the study of A-F stars, like δ Scuti stars and/or hybrid stars which are being observed by the space missions. Not only it provides a direct estimate of the mean density of the stars, but also an estimate of the frequency of the fundamental radial mode. Up to date, this latter asteroseismic observable has not been fully exploited in δ Scuti stars (with the exception of HADS, see e.g. Poretti et al. 2005; Suárez et al. 2007) because of the well-known difficulties for the mode identification.

Therefore the present results also represents an additional help for the mode identification for this type of pulsator.

The strength of this diagnostic tool is that it is almost model independent, since all the models contained in the heterogeneous dataset follow the same trend. In the following sections, we discuss different details of this finding and its consequences.

4. Some real examples

In order to have a first rough quality check of the diagnostic tool here presented, we applied it to some known δ Scuti stars: FG Vir, HD 174936, HD 174966, XX Pyx, HD 50870, and HD 209775 (see references in Table 2), for which the regular patterns have been found and analyzed. The comparison of the results found using Fig. 2 and/or Eq. 5 with those published in the literature are given in Table 2.

In general, predicted values are found within the uncertainties provided in the literature. As an example, a value of $52\mu\text{Hz}$ was proposed as possible large spacing for the *CoRoT* δ Scuti star HD 174936 by GH09. For such a value of the large spacing, and assuming no error on this value, a mean density of about $\rho = 0.31\text{gcm}^{-3}$ is predicted in the present work. Our results are in good agreement with those of GH09, taking into account that an uncertainty in $\Delta\nu$ of about $5\mu\text{Hz}$ was considered in the latter.

We compared the above results with an independent method for estimating the mean density of stars without making use of neither models nor asteroseismic information. More specifically we use the method that uses a color-density calibration for main sequence stars (see details in Tingley et al. 2011). This calibration is independent of factors such as age and metallicity, although it may suffer from significant error if extinction is

Table 2. Estimates of mean density and frequency of the fundamental radial mode of a list of δ Scuti stars found in the literature for which regularities in their powerspectra were associated with a large spacing. These estimates are compared with ours using the diagnostics presented in this work. From the left to right, columns represent the star’s identification, the observed $\Delta\nu$, its uncertainty u , determinations of the mean density $\bar{\rho}$ with their corresponding errors $\sigma(\bar{\rho})$, and determinations of the fundamental radial mode frequency. Quantities in parentheses were calculated in this work. The last three columns give the estimates of the mean density of each star using the Tingley’s method, together with their errors, and average $J - K$ magnitude differences used for its calibration.

No.	Star (μHz)	$\Delta\nu$ (μHz)	u (gcm^{-3})	$\bar{\rho}$ (gcm^{-3})	$\sigma(\bar{\rho})$ (μHz)	ν_0 (gcm^{-3})	$\bar{\rho}^T$ (gcm^{-3})	$\sigma(\bar{\rho}^T)$ (mag)	$< J - K >$
1	HD 50870 ⁶	45 ^b	5	- (0.20)	- (0.17)	80.09(162.352)	0.68	2.55	0.14
2	FG Vir ²	46 ^a	...	0.16(0.24)	0.10(-)	140.62(157.05)	0.78	2.54	0.17
3	HD 209775 ⁵	50 ^a	...	0.25 (0.29)	- (0.17)	... (168.55)	0.53	2.56	0.09
4	HD 174936 ³	52 ^b	5	0.33(0.31)	0.10(0.17)	... (174.72)	0.52	2.56	0.09
5	XX Pyx ⁴	54 ^b	2.3	0.25(0.33)	0.02(0.12)	... (180.87)	0.98	2.55	0.23
6	HD 174966 ¹	65 ^b	1	0.51(0.48)	0.11(0.17)	200.23 (212.14)	0.59	2.56	0.11

References. (a) Histograms of frequency differences; (b) Fourier Transform of the oscillation powerspectrum; (1) García Hernández et al. (2013); (2) Breger et al. (2009); (3) GH09; (4) Handler et al. (1997); (5) Matthews (2007); (6) Mantegazza et al. (2012).

unknown or poorly estimated. The method applied to our selected stars yields the values given in Table 2. The uncertainties in the density measurements are significantly larger than those obtained by asteroseismology (works cited in Table 2) and those obtained in the present work. This prevent us to conclude anything about a possible trend between the density and the large separation.

In any case, to date, the number of A-F stars for which an in-depth study of their regularities have been performed is very small. Fortunately, thanks to the asteroseismic space missions, the number of observed A-F stars with high precision increases day by day. With this diagnostic tool it will be possible to better study asteroseismic properties of A-F stars, including the large spacing itself.

5. Conclusions

We have studied the theoretical properties of the regular spacings in the oscillation spectra of δ Scuti stars. A comprehensive dataset of models representative of A-F stars covering a wide range of stellar physical magnitudes (e.g. effective temperature, gravity, metallicity, etc.) and model parameters has been performed. We have analyzed the behavior of the predicted large spacings (calculated in the intermediate frequency domain, i.e. the frequency domain in which δ Scuti stars pulsate) and their possible dependencies upon other physical magnitudes and/or model parameters. The work-flow has entirely been done using TOUCAN, a virtual observatory tool for managing and analyzing asteroseismic models that we have developed within the Spanish Virtual Observatory (see Appendix B).

Firstly, we have shown that, for the frequency domain where δ Scuti stars pulsate, the regular spacings obtained using the FT technique (as in GH09) and the regular mathematical definition of large separation computed by TOUCAN, are only equivalent when selecting oscillation modes with radial order $n \geq 1$.

Secondly, we have found a linear relation between large spacing and the mean density (in a logarithmic scale), for models in the main sequence, whatever the metallicity, mass or convection parameters (α_{ML} and d_{OV}) considered (within the values typically used for δ Scuti stars). With the use of a large model grid, we have constructed a diagnostic diagram whose properties have also been studied in detail. In particular, the most im-

portant source of error in the determination of the mean density, and other quantities like the frequency of the fundamental radial mode, ν_0 , comes from the uncertainty in the large spacing measure for rotating stars. In particular our results are valid only for stars rotating at most at 40% of the break-up velocity. Nevertheless, these uncertainties are expected to be drastically lowered by the inclusion of additional observational constraints (as shown by GH09).

Finally we have applied the diagnostic method presented in this work to five stars for which regular patterns have been found. Our estimates for the mean density and the frequency of the fundamental radial mode match reasonably (within 20% of deviation) with those given in the literature using asteroseismology. The comparison with other independent (non-asteroseismic) methods to obtain the mean density of stars reveals that asteroseismology provides, by now, the more precise estimates.

The present work implies a significant step to approach the asteroseismic studies of A-F, pulsating stars, to the level of precision achieved in solar-like stars. This is particularly relevant, not only to understand the structure and evolution of these stars but also for the study of planetary systems. Moreover, the observation of solar-like oscillations in star hosting planets provides a significant help for the characterization of the planetary systems, through the precise knowledge of the mass, radius, mean density and age of the host star (e.g. Christensen-Dalsgaard et al. 2010). The present work permits the extent of those capabilities to the study of planets orbiting around A-F pulsating stars. In fact, most of the planets discovered by direct imaging are found to be orbiting such stars. These systems are critical to understand the spin-orbital interactions between the planets and the hosting star (Winn et al. 2009; Wright et al. 2011). For these, an estimate of the age of the system is crucial to determine the mass of the planets discovered. Thanks to the large amount of A-F stars observed by CoRoT and Kepler, the number of applications of the relations here derived open a new and important window to study the structure and evolution of these stars (Uytterhoeven et al. 2011; Moya et al. 2010).

In a forthcoming study, we examine the statistical properties of the frequency spectra by analysing their autocorrelation functions and by looking at the cumulative distribution functions of the frequency separations. Results on mode visibility (Reese et al. 2013) together with the use of multi-colour pho-

tometry will help us to better assess how large separation is affected by rotation, and to improve the accuracy of the relation found in this paper.

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References

- Antoci, V., Handler, G., Campante, T. L., et al. 2011, *Nature*, 477, 570
- Baglin, A. 2003, *Advances in Space Research*, 31, 345
- Breger, M. 2000, *Delta Scuti and Related Stars*, 210, 3
- Breger, M., Lenz, P., & Pamyatnykh, A. A. 2009, *Monthly Notices of the Royal Astronomical Society*, 396, 291
- Breger, M., Pamyatnykh, A. A., Pikall, H., & Garrido, R. 1999, *A&A*, 341, 151
- Catala, C. 2009, *Communications in Asteroseismology*, 158, 330
- Christensen-Dalsgaard, J. 1988, in *IAU Symp. 123: Advances in Helio- and Asteroseismology*, ed. J. Christensen-Dalsgaard & S. Frandsen, 295
- Christensen-Dalsgaard, J., Kjeldsen, H., Brown, T. M., et al. 2010, *ApJ*, 713, L164
- Dupret, M. A., Grigahcène, A., Garrido, R., Gabriel, M., & Scuflaire, R. 2005, *A&A*, 435, 927
- Dziembowski, W. & Krolkowski, M. 1990, *Acta Astronomica* (ISSN 0001-5237), 40, 19
- Dziembowski, W. A. & Goode, P. R. 1992, *Astrophysical Journal*, 394, 670
- García Hernández, A., Moya, A., Michel, E., et al. 2009, *Astronomy and Astrophysics*, 506, 79
- García Hernández, A., Moya, A., Michel, E., et al. 2013, *Astronomy and Astrophysics*, 559, A63
- Gilliland, R. L., Brown, T. M., Christensen-Dalsgaard, J., et al. 2010, *Publications of the Astronomical Society of the Pacific*, 122, 131
- Gough, D. O. 1990, *Progress of Seismology of the Sun and Stars*, 367, 283
- Goupil, M. J., Dupret, M. A., Samadi, R., et al. 2005, *Journal of Astrophysics and Astronomy*, 26, 249
- Handler, G. 2009, in *Stellar Pulsation: Challenges for theory and observation: Proceedings of the International Conference*. AIP Conference Proceedings, Institut für Astronomie, Türkenschanzstrasse 17, 1180 Wien, Austria, 403–409
- Handler, G., Pikall, H., O'Donoghue, D., et al. 1997, *Monthly Notices of the Royal Astronomical Society*, 286, 303
- Haynes, W. M., Lide, D. R., & Bruno, T. J. 2012, *CRC Handbook of Chemistry and Physics 2012-2013* (CRC Press LLC)
- Hernández, A. G., Pascual-Granado, J., Grigahcène, A., et al. 2013, in *Stellar Pulsations: Impact of New Instrumentation and New Insights*, ed. J. C. Suárez, R. Garrido, L. A. Balona, & J. Christensen-Dalsgaard (Berlin, Heidelberg: Springer Berlin Heidelberg), 61–65
- Jackson, S., MacGregor, K. B., & Skumanich, A. 2005, *The Astrophysical Journal Supplement Series*, 156, 245
- Kallinger, T. & Matthews, J. M. 2010, *The Astrophysical Journal Letters*, 711, L35
- Kjeldsen, H., Bedding, T. R., & Christensen-Dalsgaard, J. 2008, *The Astrophysical Journal*, 683, L175
- Lebreton, Y., Monteiro, M. J. P. F. G., Montalbán, J., et al. 2008, *Ap&SS*, 316, 1
- Lignières, F., Georgeot, B., & Ballot, J. 2010, *Astronomische Nachrichten*, 331, 1053
- Lignières, F., Rieutord, M., & Reese, D. 2006, *Astronomy and Astrophysics*, 455, 607
- MacGregor, K. B., Jackson, S., Skumanich, A., & Metcalfe, T. S. 2007, *The Astrophysical Journal*, 663, 560
- Mantegazza, L., Poretti, E., Michel, E., et al. 2012, *Astronomy and Astrophysics*, 542, 24
- Matthews, J. M. 2007, *Communications in Asteroseismology*, 150, 333
- Metcalfe, T. S., Creevey, O. L., & Christensen-Dalsgaard, J. 2009, *The Astrophysical Journal*, 699, 373
- Morel, P. 1997, *A & A Supplement series*, 124, 597
- Morel, P. & Lebreton, Y. 2008, *Ap&SS*, 316, 61
- Moya, A., Amado, P. J., Barrado, D., et al. 2010, *Monthly Notices of the Royal Astronomical Society: Letters*, 405, L81
- Moya, A., Christensen-Dalsgaard, J., Charpinet, S., et al. 2008, *Astrophysics and Space Science*, 316, 231
- Moya, A. & Garrido, R. 2008, *Ap&SS*, 316, 129
- Moya, A., Garrido, R., & Dupret, M. A. 2004, *Astronomy and Astrophysics*, 414, 1081
- Moya, A. & Rodríguez-López, C. 2010, *The Astrophysical Journal Letters*, 710, L7
- Paxton, B., Bildsten, L., Dotter, A., et al. 2011, *The Astrophysical Journal Supplement*, 192, 3
- Perryman, M. A. C. 2003, *GAIA Spectroscopy: Science and Technology*, 298, 3
- Poretti, E., Michel, E., Garrido, R., et al. 2009, *Astronomy and Astrophysics*, 506, 85
- Poretti, E., Rainer, M., Weiss, W. W., et al. 2011, *A&A*, 528, A147
- Poretti, E., Suárez, J. C., Niarchos, P. G., et al. 2005, *A&A*, 440, 1097
- Reese, D., Lignières, F., & Rieutord, M. 2006, *Astronomy and Astrophysics*, 455, 621
- Reese, D., Lignières, F., & Rieutord, M. 2008, *Astronomy and Astrophysics*, 481, 449
- Reese, D. R., Prat, V., Barban, C., van 't Veer-Menneret, C., & MacGregor, K. B. 2013, *Astronomy and Astrophysics*, 550, 77
- Reese, D. R., Thompson, M. J., MacGregor, K. B., et al. 2009, *Astronomy and Astrophysics*, 506, 183
- Soufi, F., Goupil, M. J., & Dziembowski, W. A. 1998, *A&A*, 334, 911
- Suárez, J. C. 2010, *Lecture Notes and Essays in Astrophysics*, 4, 33
- Suárez, J. C., Bruntt, H., & Buzasi, D. 2005, *Astronomy and Astrophysics*, 438, 633
- Suárez, J. C., Garrido, R., & Moya, A. 2007, *Astronomy and Astrophysics*, 474, 961
- Suárez, J. C., Goupil, M. J., & Morel, P. 2006, *Astronomy and Astrophysics*, 449, 673
- Suárez, J. C., Goupil, M. J., Reese, D. R., et al. 2010, *The Astrophysical Journal*, 721, 537
- Tassoul, M. 1980, *Astrophysical Journal Supplement Series*, 43, 469
- Tingley, B., Bonomo, A. S., & Deeg, H. J. 2011, *The Astrophysical Journal*, 726, 112
- Uytterhoeven, K., Briquet, M., Bruntt, H., et al. 2010, *Astronomische Nachrichten*, 331, 993
- Uytterhoeven, K., Moya, A., Grigahcène, A., et al. 2011, *Astronomy and Astrophysics*, 534, 125
- Walker, G., Matthews, J., Kuschnig, R., et al. 2003, *PASP*, 115, 1023
- Winn, J. N., Johnson, J. A., Albrecht, S., et al. 2009, *The Astrophysical Journal Letters*, 703, L99
- Wright, D. J., Chené, A. N., De Cat, P., et al. 2011, *The Astrophysical Journal Letters*, 728, L20
- Zahn, J. P. 1992, *Astronomy and Astrophysics* (ISSN 0004-6361), 265, 115
- Zwintz, K., Lenz, P., Breger, M., et al. 2011, *Astronomy and Astrophysics*, 533, 133

Appendix A: Estimate of intrinsic errors of the method

In order to estimate the domain of validity of the method, it is necessary to determine how sensitive is the distribution of $(\Delta\nu, \bar{\rho})_i$ points in Fig. 2 to variations of the physical parameters used to construct the grid of models. Notice that the linear fit

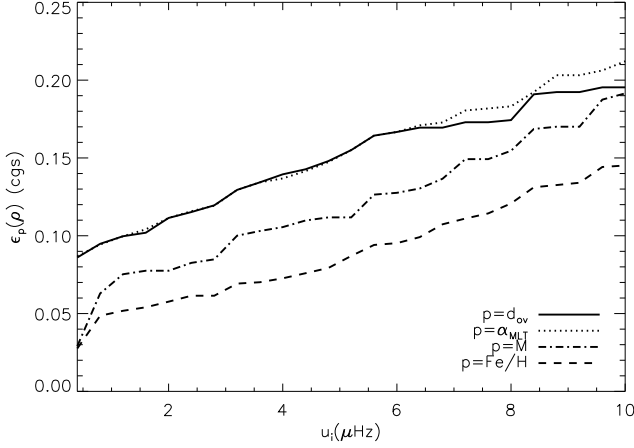


Fig. A.1. Dependence of $\sigma_p(p)(\bar{\rho})$ errors (Eq A.2) with the uncertainty in $\Delta\nu$ for the four quantities considered in this work. A value of $\Delta\nu = 60 \mu\text{Hz}$ (middle of the main sequence approximately) is assumed.

given by Eq. 5 was obtained considering each model as an independent *measurement* without uncertainties. Indeed, the whole dataset is composed by independent subsets of models: the evolutionary tracks. As a consequence, the standard procedure for calculation of coefficient errors and/or the goodness of the fit are meaningless here. Instead, we sought to estimate the errors committed by studying how variations of the physical parameters with which the model dataset was constructed (Table 1) modify the shape or the thickness of the strip shown in Fig. 2.

In particular we examined the $\Delta\nu$ - $\bar{\rho}$ relation by analyzing the maximum variation of a given parameter p at once (Table 1), leaving the remaining parameters free to vary. Then, we calculated the size of the model strip at a given value of $\Delta\nu_i$ with a given uncertainty $\pm u_i$, defined as

$$S_p(\bar{\rho}) = |\bar{\rho}(\Delta\nu_i + u_i) - \bar{\rho}(\Delta\nu_i - u_i)|, \quad (\text{A.1})$$

and the intrinsic error committed in the estimate of $\bar{\rho}$ from Fig. 2 for a given value $\Delta\nu_i$ with an uncertainty of u_i is given by

$$\epsilon_p(\bar{\rho}) = \max\{S_{p_{\max}}(\bar{\rho}), S_{p_{\min}}(\bar{\rho})\}. \quad (\text{A.2})$$

That is, we consider the maximum possible error for a given parameter variation.

These error estimates are necessarily dependent on the uncertainty in the observed value of the large spacings. We studied this dependence by calculating $\epsilon_p(x)$ for a set of u_i values (in μHz) ranging from 0 to $10 \mu\text{Hz}$. Figure A.1 shows the evolution of the errors with u_i , which is roughly linear. Notice that for $u_i \lesssim 2 \mu\text{Hz}$, values lower than 0.12 g cm^{-3} are predicted for $\epsilon_p(\bar{\rho})$. Therefore, very low uncertainties in $\Delta\nu$ are required for a good determination of the mean density. For instance, to get an uncertainty of ± 0.02 in $\bar{\rho}$, one would need to measure $\Delta\nu$ with a precision u_i lower than $1 \mu\text{Hz}$. Note that this is the precision

reached by García Hernández et al. (2013) in the study of periodicities of the δ Scuti star HD174966. Indeed, ϵ_p are intrinsic errors of the method used in this work. The total error σ (see Table 2 on the mean density depend also on all the constraints considered to model the studied star.

In the case of an ideal perfect measurement of the large spacing, the minimum precision is given by the maximum effect of the rotation (due to the star deformation) on $\Delta\nu$ (see Sect. 2.3). For very rapidly rotating objects (stars rotating faster than 40% of the keplerian velocity) the effect of rotation on the large spacing is larger than $2.3 \mu\text{Hz}$, which means an intrinsic error on $\bar{\rho}$ of approximately 0.12 g cm^{-3} . For slower stars, this intrinsic error becomes smaller. The maximum errors predicted for the estimate of the mean density range from 11% to 21% of the total variation of $\bar{\rho}$ in the main sequence.

We recall that such variations correspond to the worst case, and therefore they must be regarded as an upper limit. When other observational constraints are considered (e.g., metallicity, gravity, effective temperature, etc.), the errors in the diagnostics here proposed can drop drastically, as it was shown in GH09.

Appendix B: TOUCAN

Here we present TOUCAN, the first virtual observatory tool for asteroseismology developed by the Spanish Virtual Observatory (SVO)³, with which has been done the entire workflow of the study described in this paper. In this section we put the tool in context of necessities of the Asteroseismic community, describe its main objectives and characteristics, and detailed its current workflow. Note that such an application is constantly evolving and some of the snapshots here provided might be different in the future. In any case the main purpose and ultimate objectives will be kept.

B.1. Context

Stellar physics experiments nowadays a significant progress thanks to the rapid development of one of its main laboratories: the stellar seismology, which is the only technique allowing to probe the interior of stars for the detailed knowledge of the internal structure and the physical processes occurring there in. In the last decades we have witnessed a significant development of this technique, mainly thanks to the increase of the quantity and quality of the observations particularly from space and ground-based multisite campaigns. From space, a significant amount of high-quality asteroseismic data is available from: *MOST*⁴ (Walker et al. 2003); *CoRoT*⁵ (Baglin 2003), and *Kepler*⁶ (Gilliland et al. 2010), launched in 2009. Other missions scheduled on the near future like *GAIA* (Perryman 2003) and *PLATO* (Catala 2009), will increase by a factor of hundreds the available datasets. From the ground, dedicated photometric and spectroscopic follow-up observations for the above-mentioned space missions (e.g. Poretti et al. 2009; Uytterhoeven et al. 2010, for CoRoT and Kepler missions, respectively) are necessary for the better characterization of the stars observed by the satellites.

³ <http://svo.cab.inta-csic.es>

⁴ <http://www.astro.ubc.ca/MOST/>

⁵ Convection, Rotation, and planetary Transits. The CoRoT space mission was developed and is operated by the French space agency CNES, with participation of ESA's RSSD and Science Programmes, Austria, Belgium, Brazil, Germany, and Spain: <http://corot.oamp.fr/>

⁶ <http://kepler.nasa.gov/>

evolutionary model		oscillation model	
global	shell	general	mode
effective temperature luminosity gravity density age metallicity central hydrogen total radius total mass rotational velocity α (MLT) d_{ov} ...	temperature luminosity gravity density $d \ln T / d \ln P$ Rosseland opacity thermonuclear energy specific heat μ_e Väisälä frequency adiabatic gradient specific heat ...	F_0 F_1 F_0/F_1 Δv : large separation δv : small separation conditions v, ℓ, n ranges mode stability	n, ℓ, m v : frequency I_0 : mode inertia period Q kinetic energy phase lag δT δg NP, NG η stability ...

Fig. A.2. Overall scheme of the basic data model adopted in TOUCAN. The physical variables from equilibrium models, and the asteroseismic variables are listed on the left, and right panels, respectively.

A proper understanding of this huge amount of information requires a similar leap forward on the theoretical side (see Suárez 2010, for a recent review on this topic). Nowadays simulations of complex systems produce huge amounts of information that are difficult to manipulate, analyze, extract and publish. Significant advances have been made on this issue, e.g. the Asteroseismic Modeling Portal (AMP, Metcalfe et al. 2009) or MESA (Paxton et al. 2011) codes. However, the main problem comes from the necessity of dealing with theoretical models developed by different groups, with different codes, numerical approximation, physical definitions, etc. This lack of homogeneity makes it difficult to design automatic tools to simultaneously work with different models and/or applications able to use the models on the fly.

On the observational side, those problems have been successfully solved thanks to Virtual Observatory (hereafter, VO), which is an international initiative whose main objective is to guarantee an easy access and analysis of the information residing in astronomical archives and services. Nineteen VO projects are now funded through national and international programs, all projects working together under the IVOA⁷ to share expertise and best practices and develop common standards and infrastructures for the VO.

In this context, the Spanish VO (SVO), which joined IVOA in June 2004, is deeply involved in the development of standards that guarantee a fully interoperability between theory and observations and among theoretical collections themselves. In particular, SVO actively participated in the development of the VO access protocol for theoretical spectra⁸ and is presently working in a more general protocol called S3⁹. Examples of theoretical models published in the VO framework can be found at the SVO theoretical model server¹⁰.

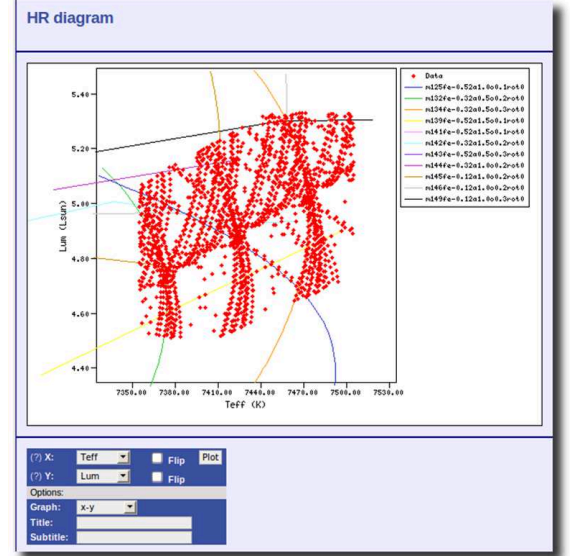


Fig. B.1. Illustration of a Hertzsprung-Russell diagram of the models matching all the input criteria simultaneously. Dots represent the effective temperature and luminosity of all the valid models. Lines represent evolutionary tracks. For clarity, only some of the models used in this work have been depicted. A colored version of the plot is accessible in the online version of the paper.

B.2. Characteristics

TOUCAN is a tool conceived to work with VO-compliant models. In the Virtual Observatory, models are described according the same data model and accessed using the same access protocol which solves all the issues regarding data discovery, data access and data representation present in non-VO tools.

The tool is intended to have a wider applicability in asteroseismology, and more generally in stellar physics and in any other field for which stellar models are required. To summarize, the main characteristics are:

⁷ <http://www.ivoa.net>

⁸ <http://ivoa.net/Documents/latest/SSA.html>

⁹ <http://ivoa.net/Documents/latest/S3TheoreticalData.html>

¹⁰ <http://svo.cab.inta-csic.es/theory/db2vo4/>

- Efficiency. TOUCAN queries multiple model databases typically in seconds.
- Collections of models are handled easily and with user-friendly web interfaces.
- The only software required is a web browser.
- Tables, figures, and model collections are fully downloadable.
- Designed for the easy and fast comparison of very different and heterogenous models.
- Visualization tools are available. Some of the plots presented in this work have been built with the TOUCAN graphic tools (see Appendix B.4).
- The tool offers new scientific potential, otherwise technically impossible or time consuming. The multivariable analysis performed in this work only required a systematic query to TOUCAN for the different parameters needed.

Furthermore, TOUCAN has also been designed for an easy and quick interpretation of the asteroseismic data coming space missions, like *MOST*, *CoRoT*, and *Kepler*, as well as future missions like the PLANetary Transits and Oscillations (*PLATO*), currently M3 candidate in ESA Cosmic Vision program, or the Transiting Exoplanet Survey Satellite (*TESS*), a new NASA space mission scheduled for launch in 2017. For this purpose, the next steps in the development of TOUCAN will be:

- The inclusion of new collections of models, namely solar-like and giant-like asteroseismic models. This will be done by calculating new model datasets with our own codes, and by adapting other model databases, built with differently codes and different physics, to TOUCAN.
- Implementation of direct link between TOUCAN and other existing VO services, allowing the search for observed physical parameters stored in VO-compliant databases (namely those of the space missions), and using them as inputs in TOUCAN.

B.3. VO service

TOUCAN has been designed following the Virtual Observatory standards and requirements. This means that, in parallel to the web interface, the system can also be accessed from other VO applications using the S3 protocol to obtain in a standard way information about:

- The available combinations of evolutionary and seismological models.
- The query parameters, their physical description and the available range of values.
- The list of models that match the query criteria and their properties.
- The stellar shell structure and the oscillation spectrum for each model.

From a technical point of view, this feature is very important, since it allows the tool to work with multiple model databases, no matter where they are physically located. Moreover, this opens the possibility of interconnecting TOUCAN with existing astronomical archives, catalogs, etc., in particular those being constructed using asteroseismic space data, already in VO-compliant form.

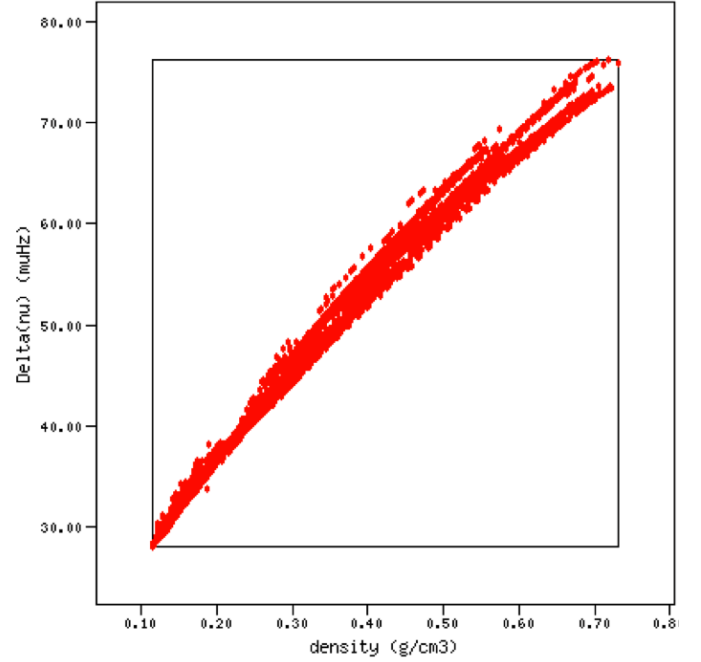


Fig. B.2. Large spacing, $\Delta\nu$, as a function of the mean density for a large set of models described in Sect. 3. Large spacings were calculated using all the frequencies available per ℓ , up to $\ell = 3$ (more details in the text). Time evolution reads from right to left, as the mean density of stars in the main sequence decrease with time. Plots were obtained using TOUCAN's graphical utilities. A colored version of the plot is accessible in the online version of the paper.

B.4. Workflow

The TOUCAN's workflow we describe here is general, and therefore applied for the present work. It is composed of three main steps :

- Input parameters specification
- Summary of the results & check out
- Model selection & online analysis

One of the most critical steps when building a tool to handle different theoretical models in a compatible way is the identification of the mandatory parameters to represent the physics involved, and their mapping into a common set of variables. In this regard, we have developed a prototype data model (Fig.A.2) for asteroseismology which contains 17 star global properties (effective temperature, surface gravity, luminosity, etc.), 44 star shell variables (density, pressure, temperature, etc.), and 35 seismic properties (frequency ranges, fundamental radial mode, large and small separation, etc.). For a maximum interoperability, we used the most common definitions in the field for these variables, with the aim of setting the basis of VO standards for asteroseismology.

Once the model parameters has been selected, TOUCAN queries the user-specified model database Here we use our own model database described in Sect. 2.1. The results obtained from these queries are shown to the user in different formats, with the possibility of managing them and, more importantly, of using TOUCAN's online graphic tools which allow the researcher to easily do *online asteroseismology*, for instance by

- Visually examining the resulting models, with some statistics and sorting possibilities for an efficient handling of the results.
- Selecting individual or multiple files to be analyzed (including shell variables analysis for equilibrium models) with the graphic tools, which allows the user to download the generated plots (an example of HR diagram build with TOUCAN's graphic tools is shown in Fig. B.1).
- Allowing the user to select individual or multiple files to be downloaded (e.g. complete evolutionary tracks) in original codes' output formats and in VOTable formats. This provides compatibility with other VO-compliant visualization tools like TOPCAT¹¹.
- To download plots in "png" format, by placing the mouse pointer on the plot window and clicking the mouse's right button.

All these characteristics make it possible to easily perform quick *on-the-fly* analysis of large set of models, and/or comparisons of different and heterogeneous models, or even model collections.

Moreover, those tools allow the user to perform statistical works on theoretical properties of multiple variables at the same time. The present work is an example of such a work: the relation between the large spacings and the mean density for δ Scuti stars shown in the contour plot (Fig. 2) was built using the data obtained from Fig. B.2 during this research workflow.

¹¹ <http://www.star.bris.ac.uk/mbt/topcat/>